Dynamical features of a classically chaotic quantum system: Symmetry breaking and the disappearance of the squeezing effect

Rui-hua Xie [1–3] and Gong-ou Xu [1]
[1]*Department of Physics, Nanjing University, Nanjing 210008, People's Republic of China*
[2]*National Laboratory of Solid State Microstructures and Instutute for Solid State Physics, Nanjing University, Nanjing 210008, People's Republic of China*
[3]*China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*
(Received 24 July 1995)

Dynamical symmetry and squeezing effect have been investigated in a classically chaotic quantum system [M. Kuś, Phys. Rev. Lett. **54**, 1343 (1985)] when it is restricted to the following initial condition: the atom in its SU(2) coherent state and the field in the vacuum state. In the regime of classically regular motion, dynamical symmetry and squeezing effects have been exposed under proper choice of the atomic coherent state; but, in that of classically chaotic motion, the symmetry is destroyed thoroughly and the squeezing effect disappears. [S1063-651X(96)07107-3]

PACS number(s): 05.45.+b, 42.50.Dv, 42.50.Lc, 32.80−t

The recently gained increase of understanding of classical Hamiltonian systems, which are nonintegrable and therefore display chaotic dynamical behaviors [1], has led to the natural question of what the quantum properties of such systems might be [2]. This field of study has been termed as ''quantum chaos.'' A physically interesting example, which belongs to this field, is the most basic model of quantum optics [3], i.e., a two-level system consisting of a single two-level atom interacting with a single quantized mode of the electromagnetic field. The Hamiltonian of this system reads

$$H=\Omega a^{\dagger}a+\omega S_z+G(a^{\dagger}+a)(S_{+}+S_{-}), \tag{1}$$

where S_z and $S_{\pm}$ are operators of the atomic inversion and transition, respectively; ω is the atomic transition frequency; $a^{\dagger}$ and a are the creation and annihilation operators of the field mode with frequency Ω, respectively; and G is the atom-field coupling constant. Throughout we employ the unit with $\hbar=c=1$. The classical limit of this model is nonintegrable and can exhibit chaotic dynamical behaviors for a large coupling constant [4]. Kuś *et al.* [5] have investigated several statistical properties of the energy levels of this model, and they found that the nearest-level spacing was highly correlated and regular in contrast to the chaotic behaviors presented in the corresponding classical version of this model. This result is inconsistent with the common belief (tested for certain variety of models) [6]: a quantum system, whose classical counterpart is chaotic, should display the spectral statistics consistent with the nearest neighbor spacing distribution (basically close to the Wigner one) for the Gaussian orthogonal ensemble (GOE) of random matrices. Graham and Höhnerbach [7] have indicated that such results were closely related to the single two-level atom being considered: with a number of two-level atoms the statistical behaviors of the GOE can be given under appropriate conditions. Recently, we have given the reason why the GOE statistics cannot appear in this system [8]: the dynamical condition for the GOE is not satisfied in the spectral statistics for this system, and the classical counterpart of this system does not properly consider the quantum-classical correspondence. However, although some regularities of the statistical spectral of the energy levels have been found in this system, Graham and Höhnerbach [9] have shown that for sufficiently strong coupling, the occupation probabilities of the two levels show irregular behaviors, more precisely, they are quasiperiodically, involving a large number of incommensurate frequencies. For small coupling, the system behaves rather regularly and the occupation probabilities show periodic ''revivals.'' In addition, in order to study the dynamics of the model (1) in the usual arena of classical dynamics, that is, in phase space, some authors [10] have done work on this topic from the point of view of the Husimi distributions. In this paper, we turn our attention to the dynamical symmetry in the model (1).

The squeezed states [11], which fulfill the uncertainty relation with a reduced quantum dispersion, have been an interesting topic due to their potential application [12] in gravity wave detection, high-resolution spectroscopy, quantum nondemolition experiments, quantum communications, and low-light-level microscopy. It has been shown both theoretically [13] and experimentally [14] that a squeezed field can be generated by various physical processes. Meanwhile, increased attention has also been paid to the squeezing of quantum fluctuations of the atomic dipole variables (i.e., the atomic squeezing) [15]. Moreover, the relationship between the field and atomic squeezing has even been discussed by Wodkiewicz *et al.* [16]. However, less work has been done on this topic in the classically chaotic system. It forms another aim of this paper.

In the present paper, we have exposed the following additional dynamical features in the model (1): (a) there exist a striking dynamical symmetry and the field and atomic squeezing in the case of sufficiently weaker coupling (i.e., the regime of classically regular motion [4]) when the system is restricted to the following initial condition: the atom in its SU(2) coherent state and the field in the vacuum state

$$|\psi(0)\rangle=\sin\left(\frac{\theta}{2}\right)e^{-i(\delta/2)}\left|-\frac{1}{2},0\right\rangle+\cos\left(\frac{\theta}{2}\right)e^{i(\delta/2)}\left|\frac{1}{2},0\right\rangle, \tag{2}$$

where $0 \leqslant \theta < \pi$ denotes the initial distribution of the atom, and $0 \leqslant \delta < 2\pi$ is the relative phase between the ground and excited states. (b) for sufficiently stronger coupling (i.e., in the regime of classically chaotic motion [4]), the symmetry is destroyed thoroughly and the squeezing effect disappears.

In order to investigate the squeezing properties of the radiation field and the atom, we define the slowly varying Hermitian quadrature operators $a_1 = \frac{1}{2}(ae^{i\Omega t} + a^\dagger e^{-i\Omega t})$, $a_2 = 1/2i(ae^{i\Omega t} - a^\dagger e^{-i\Omega t})$, $S_1 = \frac{1}{2}(S_+ e^{-i\omega t} + S_- e^{i\omega t})$, and $S_2 = 1/2i(S_+ e^{-i\omega t} - S_- e^{i\omega t})$, where S_1 and S_2, in fact, correspond to the dispersive and absorptive components of the slowly varying atomic dipole [17], respectively. The above operators obey the commutation relations $[a_1, a_2] = i\frac{1}{2}$ and $[S_1, S_2] = iS_z$. Correspondingly, the Heisenberg uncertainty relations are $(\Delta a_1)^2(\Delta a_2)^2 \geqslant \frac{1}{16}$ and $(\Delta S_1)^2(\Delta S_2)^2 \geqslant \frac{1}{4}\langle S_z\rangle^2$. It is convenient to define the following functions $h_i = (\Delta a_i)^2 - \frac{1}{4}$ and $F_i = (\Delta S_i)^2 - \frac{1}{2}|\langle S_z\rangle|$ $(i=1,2)$. Then,the field squeezing is defined if $h_i < 0$ ($i=1$ or 2) [13], so is the atomic squeezing if $F_i < 0$ ($i=1$ or 2) [15].

Near resonance and for sufficiently weaker atom-field coupling, the rotating-wave approximation (RWA) applies, then, the Hamiltonian (1) can be written as

$$H = \Omega a^\dagger a + \omega S_z + G(a^\dagger S_- + aS_+). \tag{3}$$

This is the well-known Jaynes-Cummings model [19], which can be solved exactly. Restricting to the initial condition (2), we have obtained the time evolution F_1 and h_2 as follows:

$$F_1(t) = \frac{1}{4} - \frac{1}{4}\sin^2(\theta)\cos^2(Gt)\cos^2(\delta) - \left|\frac{1}{2}\cos^2\left(\frac{\theta}{2}\right)\cos^2(Gt) - \frac{1}{4}\right|, \tag{4}$$

$$h_2(t) = \frac{1}{2}\cos^2\left(\frac{\theta}{2}\right)\sin^2(Gt) - \frac{1}{4}\sin^2(\theta)\sin^2(Gt)\cos^2(\delta). \tag{5}$$

Then we have verified that a dynamical symmetry between F_1 and h_2 is exposed under the parameter condition: arbitrary phase δ and $-1 < \cos(\theta) \leqslant 0$. Furthermore, we have found that under the following parameter condition: $-1 < \tan(\delta) < 1$ and $-1 < \cos(\theta) < -\tan^2(\delta)$, the fluctuations in S_1 and a_2 can be squeezed almost at all times, with identical squeeze duration ($GT_S = \pi$) and squeeze period ($GT_P = \pi$) and maximum height of squeeze peak $A_{\max} = |F_1 < 0|_{\max} = |h_2 < 0|_{\max} = \cos^2(\theta/2)[\sin^2(\theta/2)\cos^2(\delta) - \frac{1}{2}]$ [$A_{\max}$ appearing at $Gt = k\pi$ and $(k+1/2)\pi$ for atomic squeezing and field squeezing, respectively, throughout the paper, $k=0$, integer], but out of phase (i.e., $\pi/2$). It is clear that there exists a striking dynamical symmetry between the field and atomic squeezing (SFAS), and here the squeezed atom can radiate a squeezed field almost at all times.

Now we examine the quantum dynamical properties of the non-RWA-Hamiltonian (1). Taking $|m,n\rangle$ as a basis, where $S_z|m,n\rangle = m|m,n\rangle$, $m = \pm\frac{1}{2}$ and $a^+ a|m,n\rangle = n|m,n\rangle$ ($n=0$, integer), we can obtain the eigenstate $|\phi_i\rangle$ and the energy eigenvalue E_i ($i=1,2,\ldots$) of the non-RWA-Hamiltonian (1) by truncating an infinite matrix to finite order [5,7–9]. Given the initial state $|\psi(0)\rangle$ of the system, the expectable value of an observable variable η at time t can be calculated as follows:

$$\langle\eta\rangle = \langle\psi(t)|\eta|\psi(t)\rangle = \langle\psi(0)|e^{iHt}\eta e^{-iHt}|\psi(0)\rangle = \sum_i \sum_j \langle\psi(0)|\phi_i\rangle\langle\phi_j|\psi(0)\rangle\langle\phi_i|\eta|\phi_j\rangle e^{-i(E_j - E_i)t}. \tag{6}$$

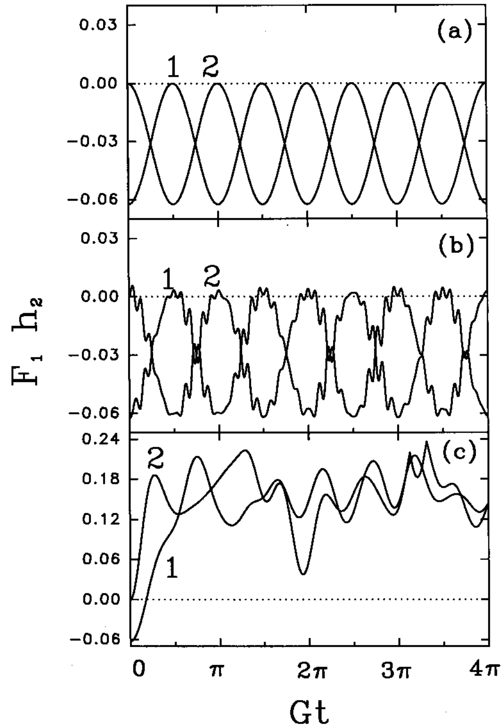

FIG. 1. Time evolution of F_1 (the line marked "1") and h_2 (the line marked "2") in the non-RWA-Hamiltonian (1) for $\theta = 2\pi/3$, $\delta = 0$, $\Omega = \omega = 1$. (a) $G = 10^{-5}$; (b) $G = 10^{-1}$; (c) $G = 1$.

(I) $\theta = 2\pi/3$ and $\delta = 0$. For a sufficiently weaker coupling: Fig. 1(a) shows the time evolution behaviors of F_1 and h_2 for $\theta = 2\pi/3$, $\delta = 0$, and $G = 10^{-5}$. We find that there exists a SFAS, where $GT_S = GT_P = \pi$, and $A_{\max} \approx 0.062$ for the field and atomic squeezing, respectively, appears at the time $Gt = (k + \frac{1}{2})\pi$ and $k\pi$. This result can be understood as addressed above: in this case, the RWA applies, hence the SFAS is observed in Fig. 1(a).

For a strong coupling: when the coupling constant G is increased up to the order 10^{-1}, we notice from Fig. 1(b) that GT_S and GT_P are smaller than the constant π, $A_{\max}$ is less than 0.062, and the squeezed atom cannot radiate a squeezed field any more at the time regions near $Gt = k\pi$. It is clear that the SFAS begins to be destroyed for the coupling constant G up to the order 10^{-1}. Furthermore, for sufficiently stronger coupling (i.e., in the regime of classically chaotic motion [4]), as shown in Fig. 1(c), the squeezing disappears almost entirely (only atomic squeezing appears at the early time regions), and the SFAS is destroyed thoroughly. In the RWA Hamiltonian (3), the counter-rotating wave terms are neglected, but it is switched on in the non-RWA-

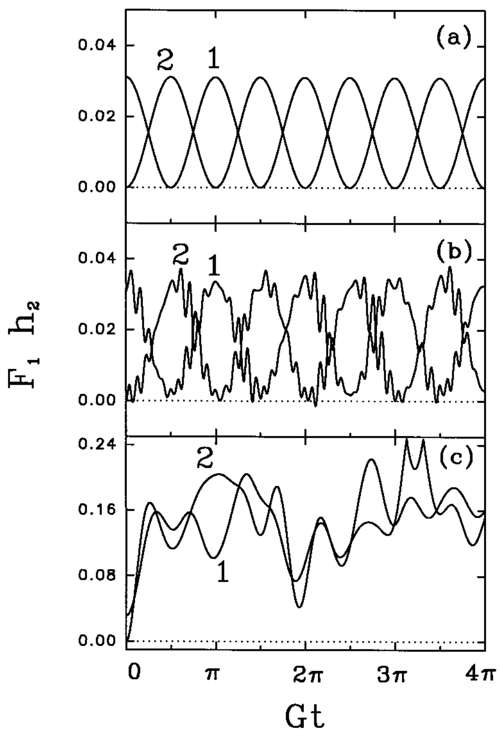

FIG. 2. For $\theta=2\pi/3$ and $\delta=\pi/4$, the others are the same as for Fig. 1.

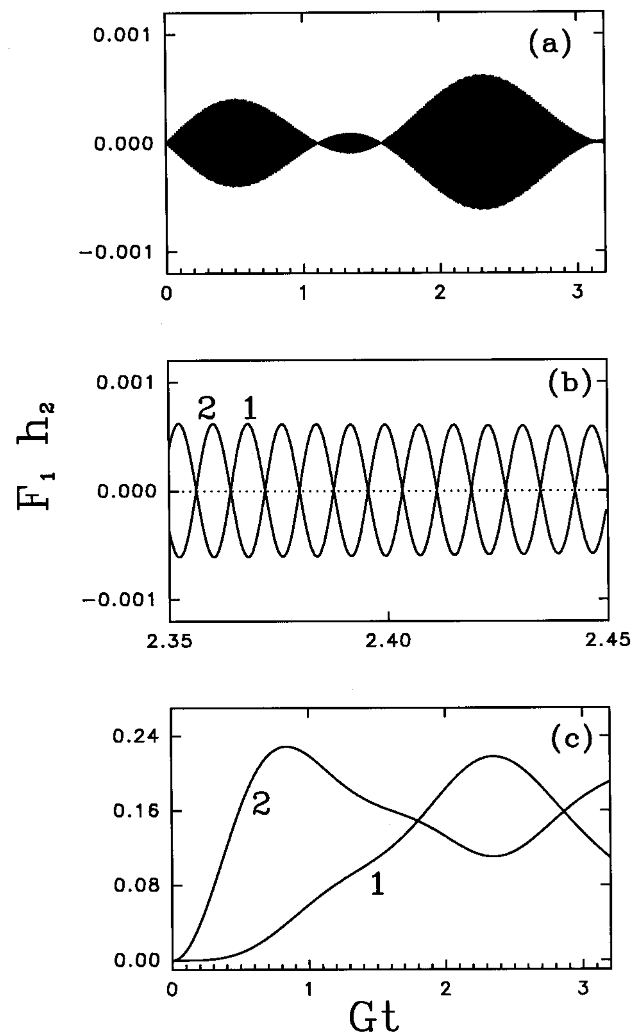

FIG. 3. Time evolution of F_1 (the line marked ''1'') and h_2 (the line marked ''2'') in the non-RWA-Hamiltonian (1) for $\theta=\pi/2$, $\delta=0$, $\Omega=\omega=1$. (a) $G=5\times10^{-3}$; (b) during the time $2.35<Gt<2.45$ in case (a); (c) $G=1$.

Hamiltonian (1). Therefore it is the interference between the real-photon processes and the virtual-photon processes that destroys the SFAS.

(II) $\theta=2\pi/3$ and $\delta=\pi/4$. In Fig. 2, we show the time evolution behaviors of F_1 and h_2 for $\theta=2\pi/3$ and $\delta=\pi/4$ and with the same coupling constant as for Fig. 1. Also, we find for a sufficiently weaker coupling, as shown in Fig. 2(a), there exists a dynamical symmetry between F_1 and h_2; the symmetry begins to be destroyed for the coupling constant up to the order 10^{-1} [see Fig. 2(b)]; and the symmetry is destroyed thoroughly for sufficiently stronger coupling [see Fig. 2(c)].

(III) $\theta=\pi/2$ and $\delta=0$. According to Eqs. (4) and (5), it is obvious that in the RWA Hamiltonian (3), both F_1 and h_2 equal zero for $\theta=\pi/2$ and $\delta=0$. In the non-RWA-Hamiltonian (1), however, we find that for a sufficiently weaker coupling, as shown in Fig. 3(a), there appear quantum collapse revivals, and the squeezing effect is observed at the collapse regions. In detail, during the time $2.35<Gt<2.45$ in case (a), we notice from Fig. 3(b) that there also exists a dynamical symmetry between F_1 and h_2. For a sufficiently stronger coupling, as shown in Fig. 3(c), we find that the quantum revivals and the squeezing effect disappear and the dynamical symmetry is destroyed.

(IV) Conclusions and discussions. We draw conclusions as follows: (a) under the proper choice of an initial atomic coherent state, dynamical symmetry and squeezing effects are exposed in the regime of classically regular motion; (b) however, in the regime of classically chaotic motion, the squeezing effect disappears and the symmetry is destroyed. thoroughly.

The sufficiently weaker coupling constants taken above are accessible in the micromaser experiment [19], and the initial condition (2) can be realized in the laboratory [16]. This means that our above results for sufficiently weaker couplings are significant in the micromaser experiment.

For a strong coupling constant, the two-level approximation breaks down far sooner than the RWA, and other atomic levels, which are coupled by the field to the two levels with a comparable strength, will cause an impact on the dynamics which is at least comparable to the corrections of the counter-rotating wave terms to the RWA. In the light of this point, the studies for strong coupling are not realistic in quantum optics and micromaser experiments. We feel, however, that the non-RWA-Hamiltonian (1), as addressed before, may be of its own academic interest [4,5,7–10] in the field of chaos. In Ref. [9], Graham and Höhnerbach have indicated that the appearance of chaos in the full classical system is signaled, in the quantum system, by the disappearance of the quantum revivals. Here we obtained a similar conclusion and even found some additional dynamical features, i.e., symmetry breaking and disappearance of squeezing effect.

The model (1) is, in fact, a very popular model in physics, with a large variety of applications in condensed matter physics [20], macroscopic quantum tunneling [21], atomic and molecular physics [22], as well as quantum optics [18] and quantum chaos [5,7–10] (note: in its different fields of applications, the Hamiltonian (1) bears different names, such as, ''molecular polaron model,'' ''Rabi Hamiltonian,'' etc.). In these contexts, the coupling constant is intrinsically much larger than that in quantum optics. So, our findings may have physical impact on these contexts.

ACKNOWLEDGMENTS

This work was supported by the National Basic Research Project, ''Nonlinear Sciences,'' China. One of us (R.-H.X.) was given financial support by the Tianma Microelectronics Co. Ltd in Shenzhen, China.

---

[1] Z. Qu, G. Hu, G. Yang, and G. Qin, Phys. Rev. Lett. **74**, 1736 (1995), and references therein.

[2] M.C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, New York, 1990); F. Haake, *Quantum Signatures of Chaos* (Springer-Verlag, New York, 1991).

[3] L. Allen and J.H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).

[4] P.I. Belobrov, G.M. Zaslavski, and G.Kh. Tartakowski, Zh. Éksp. Teor. Fiz. **71**, 1799 (1976) [Sov. Phys. JETP **44**, 945 (1976)]; P.W. Milonni, J.R. Ackerhalt, and H.W. Galbraith, Phys. Rev. Lett. **50**, 966 (1983); R.P. Frueholz, and J.C. Camparo, Phys. Rev. A **47**, 4404 (1993), and references therein.

[5] M. Kus, Phys. Rev. Lett. **54**, 1343 (1985); W.-H. Steeb, C.M. Villet, and A. Kunick, Phys. Rev. A **32**, 1232 (1985).

[6] F. Leyvraz and T.H. Seligman, Phys. Lett. A **168**, 348 (1986).

[7] R. Graham and M. Höhnerbach, Phys. Rev. Lett. **57**, 1378 (1986).

[8] R.H. Xie, D.H. Liu, and G.O. Xu, Z. Phys. B **99**, 605 (1996).

[9] R. Graham and M. Höhnerbach, Z. Phys. B **57**, 233 (1984).

[10] M. Cibils *et al.*, Phys. Rev. A **46**, 4560 (1992); L. Müller *et al.*, *ibid.* **44**, 1022 (1991); M. Aguiar *et al.*, Europhys. Lett. **15**, 125 (1991).

[11] J. Opt. Soc. Am. B **4**, 1453 (1987), Special Issues on the squeezed state, edited by H. J. Kimble and D. F. Walls.

[12] Appl. Phys. B, **55**, 189 (1992), Special Issues on its applications, edited by E. Giacobino and C. Fabre.

[13] J. Mod. Opt. **34**, 709 (1987), Special Issues on the squeezed field, edited by P. L. Knight.

[14] A.M. Fox *et al.*, Phys. Rev. Lett. **74**, 1728 (1995), and references therein.

[15] T. Nasreen and M.S.K. Razmi, Phys. Rev. A. **46**, 4161 (1992), and references therein.

[16] K. Wodkiewicz *et al.*, Phys. Rev. A **35**, 2567 (1987), and references therein.

[17] N.B. Narozhny, J.J. Sanchez-Mondragon, and J.H. Eberly, Phys. Rev. A **23**, 236 (1981).

[18] E.T. Jaynes and F.W. Cummings, Proc. IEEE **51**, 126 (1963).

[19] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987); G. Rempe, F. Schmidt-Kaler, and H. Walther, *ibid.* **64**, 2783 (1990); O. Benson, G. Raithel, H. Walther, *ibid.* **72**, 3506 (1994).

[20] G. Yuval and P.W. Anderson, Phys. Rev. B **1**, 1522 (1970); R. Beck, W. Götz and P. Prelovsek, Phys. Rev. A **20**, 1140 (1979).

[21] A.O. Caldeira and A.J. Leggett, Phys. Rev. Lett. **46**, 211 (1981); H. Spohn and R. Dumcke, J. Stat. Phys. **41**, 389 (1985).

[22] D. Feinberg and J. Ranninger, Physica D **14**, 29 (1984).